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ABSTRACT

A model to estimate higher education enrollments and cost effects of financial aid plans is presented. A combination of the freshman demand and transition models allows the prediction of high school graduates who enter colleges, complete sequential levels of education, and complete degree programs. The freshman demand model is adapted from that of Radner and Miller. Six income intervals are used to calculate freshman demand. Policy plans involving work study payments, student loans, and student grants are analyzed using the model. Plans of the Carnegie Council on Policy Studies in Higher Education, a plan of the Committee on Economic Development, and a variant of these plans are used. The cost of each plan per student in each income group attending public and private higher education institutions are calculated. All of the plans increase the demand for higher education; comparison of the plans are presented. A model for forecasting student enrollments in higher education institutions developed by the National Commission on the Financing of Postsecondary Education is critiqued and compared to the model used in this study. (SW)

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Enrollment and Cost Effects of Financial Aid Plans for Higher Education

A Report for the Carnegie Council on Policy Studies in Higher Education

Joseph Hurd

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Project on Quantitative Policy Analysis Models of Demand and Supply in Higher Education

Technical Report No. 1

July, 1977

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ENROLLMENT AND COST EFFECTS OF FINANCIAL AID PLANS FOR HIGHER EDUCATION

Joseph Hurd

Technical Report No. 1

July, 1977

Carnegie Council on Policy Studies

in Higher Education

Project on Quantitative Policy Analysis Models

of Demand and Supply in Higher Education

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Contents

I. Introduction and Summary of Results 1

II. Brief Description of the New Model 7

III. Use of the Model: Enrollment Effects 9

IV. Use of the Model: The Costs of the Plans 34

V. Critique of the National Commission on the Financing of Postsecondary Education Model 43

VI. A Closer Look at the Model 51

VII. Conclusion 60

VIII. Appendices 64

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I. Introduction and Summary

Our society has placed and continues to place great importance on college and university training. We have long thought that a college degree was the guarantor of a life of relative affluence and prestige, since casual observation has revealed the correlation between income (monetary and non-monetary) and education. To the middle and lower classes, higher education has been one key to advancement and status. To them a college degree has been the certificate of having "made it."

One aspect of the relation between higher education and income has been the positive correlation between family income and higher education attendance. As family income increases the frequency of attending college increases. Because education has been viewed as an important vehicle for upward mobility, the unequal relationship between income and education has spurred interested parties to make numerous policy proposals concerning financial aid to students.[1]

Although plenty of student aid policy recommendations have been made, until recently the facts needed to choose among them have been crude or nonexistent. One wants to know how, and by how much, a change in the cost of attending a college or university will change enrollment demand. And one wants to know how much the new enrollments will add to the cost of an aid plan. Without estimates of what might be called enrollment effects it is difficult to choose between contending aid proposals where the intention of the proposal is to change the relationship between higher education attendance and family income. Without

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[1] A compilation of the proposals and positions of nineteen groups and associations can be found in College Entrance and Examining Board, 1975.

estimates of the enrollment effects it is difficult, and probably imprudent, to consider adopting an ambitious aid program which could induce large, unanticipated financial obligations on the providers of the aid and on the colleges and universities themselves.

There is probably no single definition of equality of opportunity in higher education upon which all interested parties would agreed. We have chosen to define equality of opportunity as follows: For students of a given level of academic ability, attendance at institutions of higher education (IHE) shall be the same regardless of family income. To be sure, we cannot directly quantify academic ability. Instead, we use Scholastic Aptitude Test (SAT) scores as a measure of it.

We have developed a model of sufficient generality to estimate the enrollment effects of alternative policy changes in the cost of the policy plan under consideration. Our model is such that enrollment effects and plan costs (of policy plans that typically base changes in tuition levels or in student grants on family income) can be calculated so that the impact of the plan on various income groups or academic ability levels can be isolated.

Section II of this report is a short verbal description of our model. The section, though brief, is adequate for understanding the results in Sections III and IV.

In Sections III and IV we consider five policy plans. Section III presents these plans and their calculated enrollment effects. Section IV discusses the costs of each plan. By cost we mean both aid distri-

buted to students and the subsidy to students which results from tuition being less than the average cost of instruction.

Three policies we have chosen to analyze are two versions of the program outlined by the Carnegie Council on Policy Studies in Higher Education and a proposal by the Committee for Economic Development. We also have included two variants of a plan of our own that equalizes opportunity for given academic ability, irrespective of family income, for all income groups up to and including a given level of income. Without intending to be pretentious, we call these the Equal Opportunity plans.

We find that two of the plans, the CED plan and Equal Opportunity Plan I, achieve roughly similar results in terms of enrollment and overall cost. Both come close to reaching the goal of equality of opportunity in higher education. Both plans, either through aid alone in the case of the Equal Opportunity plan, or through aid and public tuition increases in the case of the CED plan, improve the ability of privately controlled IHEs to compete for students. When fully funded and operating over time, both plans would cost less than the other plans--the CED plan about 2.7 billion and the Equal Opportunity plan about 3.2 billion 1974 dollars per year. If public tuitions were to rise enough to cover half the cost of instruction (as in the CED plan), the aid cost of Equal Opportunity Plan I would increase from $2.7 billion to roughly $4 billion.

The Carnegie plans are expensive; each would cost much more than

the CED plan. The first Carnegie plan would cost nearly one billion 1974 dollars, and the second nearly 7.3 billion 1974 dollars. Moreover, the Carnegie plans in a sense discriminate against students of middle and upper family incomes in that fewer of these groups would enroll in institutions than would lower-income students of similar ability. The Carnegie plans would increase enrollment demand substantially, particularly for students from families with the lowest incomes.

Equal Opportunity Plan II would be about as expensive as the Carnegie plans. More than the latter it would shift enrollments toward private rather than public colleges and universities and total enrollments would be less. In all other particulars the expense of Equal Opportunity Plan II simply reflects the generous increase in aid it offers over that offered by Equal Opportunity Plan I.

If we compare all the policy plans by how much each would cost in aid per new enrollment in either two- or four-year institutions, we find that the Carnegie plans are more efficient, meaning that the cost of each new enrollment is lower, than the CED plan, and that the more generous Equal Opportunity plan is by far the least efficient. The first Equal Opportunity plan falls between the first Carnegie plan and the CED plan by this comparison. We estimate the range in aid cost per new enrollment to be from about $8,842 (Carnegie) to $16,830 (Equal Opportunity plan).

Of the policy proposals that we have considered the Carnegie plans would be the easiest, given funding, to implement, since existing

facilities at each institution could continue to grant aid as they do now, only in larger amounts. Some incentive to raise tuitions would result from the fact that the State Student Incentive Grants (SSIG) cannot be higher than tuition; for colleges and universities with low tuitions and which typically attract lower income students, the increased tuition would be paid for by the SSIG.

The Equal Opportunity plans, although in principle easy to administer, would encourage tuition increases since aid subsidies would be proportional to tuition and other costs. Students from eligible income groups would spend less than a dollar for each dollar increase in tuition, particularly in the higher-cost private universities. If, as might well be the case, in the actual implementation of one of these plans a dollar ceiling were placed on aid to the student, then our analysis of these plans would be different depending on the limit to aid. If the ceiling were, say, two-thirds or three-fourths of the average cost of instruction at a highly selective, private IHE, then the results we have calculated for Equal Opportunity Plan I would be unchanged. The aid ceiling would certainly dampen the incentive for some IHEs to increase tuition.

The CED plan would not be difficult to administer, but implementation would require the publicly controlled colleges and universities to raise tuition in concert and by specific dollar amounts. How this cooperation would be achieved is not clear.

The predictive power of our model rests on the parameter estimates

and model of Radner and Miller,[2] which imply particular price elasticities of demand. Other researchers, not using the same data or model, have reported estimated elasticities greater than that of Radner and Miller.[3] If we are using an underestimate of what we can call price response, then we have underestimated the enrollments and induced costs generated by the policy plans as we have stated them, in other words, we have overstated the aid levels needed to induce the enrollment charges we report. We have investigated the effect of doubling the elasticities in our model. For example, if the true elasticities in the Carnegie plans were twice as large as we have calculated them to be, the same enrollments could be achieved with a 55-percent reduction in cost.

Section V is a critique of an antecedent model developed by the National Commission on the Financing of Postsecondary Education for the same type of policy analysis that we have done. It is followed by a comparison of our model with the older one.

In Section VI we describe our model in greater detail. Section VII presents some concluding remarks.

Two appendices are provided in Section VIII. The first should help the reader unfamiliar with Radner's and Miller's model to better understand the National Commission on the Financing of Postsecondary

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[2] R. Radner, L. Miller; Demand and Supply in U.S. Higher Education, McGraw-Hill, 1975.

[3] G. Weathersby, Gregory Jackson; Individual Demand for Higher Education, 1974.

Education model as well as ours. The second appendix is related to National Commission calculations of interest to the enthusiast, although perhaps not to the general reader. Both appendices are referred to in the text.

The reader who is mainly interested in the results rather than the methods of our analysis might wish to restrict his reading to Sections II, III, IV, and VII.

II. Brief Description of the New Model

Our model for forecasting the demand for higher education is made up of two components, the first of which we call the freshman demand model and the second of which we call the transition model. The freshman demand model is adapted from that of Radner and Miller. Using their parameter estimates, the fitted values, which depend on policy change in the cost of attendance, become the predicted proportions of high school graduates attending various kinds of institutions or no institutions at all. We call these proportions the predicted freshman demand frequencies.

The transition model uses the freshman demand frequencies to predict subsequent (sophomore and junior, for example) undergraduate enrollment proportions. The transition model is comprised of transition states, as defined in Table 1, and transition probabilities, which are the chances of moving from one transition state to another. The transition states distinguish between persons enrolled at various levels in IHEs, persons who have "stopped out," and persons who definitely have dropped out of higher education. The transition probabilities have been estimated judgmentally from attrition and persistence data and adjusted so that the predicted enrollments are in rough agreement with enrollment projections made by the National Center for Educational Statistics.

Combining the freshman demand and transition models enables us to predict the proportion of high school graduates who enter colleges and universities, complete sequential levels of education, and complete degree programs. We apply the predicted enrollment proportions to known or projected numbers of high school graduates to predict actual enrollments.

TABLE 1

Definition and Discussion of States in the Transition Model

State 1 -- Neither enrolled in an IHE nor allowed to enroll in the future. A heterogeneous state, since it includes both students who have obtained a bachelor's degree and students who will never obtain a degree.

State 2 -- Enrolled in the first year of a two-year IHE. This state represents first-time two-year IHE enrollments.

State 3 -- Enrolled in the second year of a two-year IHE.

State 4 -- Enrolled in the first year of a four-year IHE. This state represents first-time enrollments in four-year IHEs.

State 5 -- Enrolled in the second year of a four-year IHE.

State 6 -- Enrolled in the third year of a four-year IHE.

State 7 -- Enrolled in the fourth year of a four-year IHE.

State 8 -- Enrolled in the fifth year of a four-year IHE. This state takes into account five-year degree programs.

State 9 -- Not enrolled but eligible to enroll for the first year of an IHE.

State 10 -- Not enrolled but eligible to enroll in the second year of an IHE.

State 11 -- Not enrolled but eligible to enroll in the third year of an IHE.

State 12 -- Not enrolled but eligible to enroll in the fourth year of an IHE.

State 13 --Not enrolled but eligible to enroll in the fifth year of an IHE. In states 9 through 13, the level of eligible enrollment indicates past completion of from zero to four years of postsecondary education.

III. Use of the Model: Enrollment Effects

In calculating the baseline undergraduate enrollments and the subsequent policy comparisons, we have chosen to consider six income intervals, each of which is represented by an interior point in calculating freshman demand. The greater the number of intervals the better the approximation would be, but the cost and complexity of the computation would also increase. We consider six intervals to be sufficiently accurate for our present purposes. Each income interval represents a proportion of the population. Table 2 lists family income intervals, evaluation points, and population weights.

TABLE 2

Income (Y), Interval ($)	Evaluation Point ($)	Population Weight (%)
$T < 7,600$	6,000	22.3
$7,600 < Y \leq 10,500$	9,000	14.9
$10,500 < Y \leq 12,900$	12,000	12.6
$12,900 < Y \leq 15,500$	14,000	12.1
$15,500 < Y \leq 22,000$	19,000	15.7
$22,000 < Y$	25,000	22.2

Until this point we have used the term enrollment without qualification, as if its meaning was clear. This is not the case. Among nongraduate students are part and full-time students enrolled in course work that may or may not be creditable toward a bachelor's or associate

degree. In the discussion below we use the term enrollment to mean full-time equivalent (FTE), degree-credit enrollment. (The single exception to this definition is that for first-time two-year IHE enrollments. In this case the term includes full-time plus part-time students.)

To calibrate our model we use seven measurements of enrollments or degrees for the years from 1972 to 1983. We use 1966 as the year the model was initiated so that the 1972 figures reflect the past fairly.

Our baseline figures are shown in Table 3, and Table 4 gives the National Center for Educational Statistics (NCES) figures for comparison.[4] Our figures for first-time enrollments and bachelor's degrees are very close to those of the NCES. Our total enrollment stays within 5 percent of the NCES total enrollment projections, with most of the difference between the two due to our higher two-year IHE enrollment. One immediate potential source of disagreement between our calculations and those of the NCES is the use by the latter of nonlinear extrapolation, while our model calculates constant proportions through time. This difference in method implies divergence of enrollments through time, even if the starting points are close. The difference between our baseline and that of the NCES is not of great importance, however, because in general our conclusions are not sensitive to small changes in the baseline.

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[4] National Center for Educational Statistics - Projection of Educational Statistics, 83-84, 1974 edition.

TABLE 3

Baseline 1972-1983 (in thousands)

Baseline Undergraduate Enrollments, Degrees, and First-Time Enrollments

Year	Total FTE	2-Year FTE	4-Year FTE	A.A.	B.A.	First-time 2-Year	First-time 4-Year
1972	5,914	1,400	4,514	174	853	676	1,001
1973	6,031	1.424	4,606	176	880	684	1,011
1974	6,143	1,449	4,695	181	909	697	1,030
1975	6,234	1,467	4,767	183	925	704	1,039
1976	6,303	1,480	4,823	186	944	705	1,042
1977	6,362	1,491	4,872	188	956	711	1,048
1978	6,391	1,494	4,898	191	971	709	1,043
1979	6,383	1,486	4,897	191	984	699	1,028
1980	6,347	1,471	4,877	191	989	692	1,014
1981	6,287	1,452	4.835	190	993	681	1,000
1982	6,184	1,423	4,760	186	992	662	970
1983	6,034	1,381	4,652	184	981	638	928

TABLE 4

National Center for Educational Statistics

Undergraduate Degree Credit Enrollments and Degrees Awarded and First-time Enrollments (in thousands)

Year	Total FTE	2-Year FTE	4-Year FTE	B.A.	First-time 2-Year	First-time 4-Year
1972	5,896	1,197	4,699	883	676	1,001
1973	5,975	1,253	4,722	954	681	1,004
1974	5,955	1,256	4,699	977	690	1,021
1975	5,980	1,271	4,709	975	697	1,030
1976	6,036	1,297	4,739	967	699	1,033
1977	6,105	1,322	4,783	983	703	1,041
1978	6,142	1,341	4,801	1,005	699	1,035
1979	6,132	1,348	4,784	1,015	687	1,017
1980	6,102	1,348	4,754	1,029	677	1,002
1981	6,046	1,341	4,705	1,042	666	986
1982	5,943	1,323	4,620	1,043	643	953
1983	5,781	1,293	4,488	1,030	612	905

Source: National Center for Educational Statistics, 1974.

A policy plan typically chooses among four instruments to induce changes in enrollment: work-study payments, loans to students, grants to students, and raising or lowering tuition. The latter two instruments are "pure" in that they affect only the net cost of going to college. They do not alter any other present or future costs or conditions of going to college. The first two instruments provide funds to the student at a cost. A loan is truly aid only if the terms of the loan provide some subsidy compared to ordinary free-market loans. Furthermore, a loan must be repaid. Payments for work-study lower the attendance costs only if the student cannot find other employment at that wage rate. More importantly, work-study reduces the time and energy the student has to pursue his academic courses.

The policies we have chosen to analyze with our model include two versions of the program outlined by the Carnegie Council on Policy Studies in Higher Education and a proposal by the Committee for Economic Development.[5] Let us call them the Carnegie plans and the CED plan. We also include two variants of a plan implicit in the model, which equalizes opportunity for given academic ability, irrespective of family income, for all groups up to and including a given level of income. We call them the Equal Opportunity plans. The figures in all the aid proposals have been converted to 1974 dollars.

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[5] These plans can be found in the Carnegie Council on Policy Studies in Higher Education, The Federal Role in Postsecondary Education (1975); and The Committee for Economic Development, The Management and Financing of Colleges, 1973.

The Carnegie and CED plans propose work-study or student-loan program changes. In our analysis of these plans we have not calculated the effects of these changes. This is not to say that work-study and student-loan programs are unimportant or ineffective. Rather the lack of analysis reflects our ignorance of the magnitude of the secondary effects of these forms of aid on student participation and persistence.

Carnegie Plan I has three forms of aid. The first is the Basic Economic Opportunity Grant (BEOG), which pays a maximum of $1,600 to every enrolled (FTE) student with a family income of less than $7,600. Furthermore, BEOG aid is paid proportionally to enrolled students with family incomes between $7,600 and $12,900. No BEOG aid is available to students with family income greater than $12,900.

Another form of aid is the State Student Incentive Grants (SSIG). Under this program, students with a family income of less than $7,600 are given a maximum of $1,500 for instructional costs. As with BEOG, students with a family income between $7,600 and $12,900 receive a proportion of the maximum amount.

Finally, Carnegie Plan I proposes the Tuition Equalization Grant (TEG), a subsidy for attending a private school that is independent of family income. In our analysis, any student enrolling in a private IHE is awarded the standard $750 TEG. The total SSIG and TEG for any student may not exceed $1,500.

Carnegie Plan II is a simple variant of the first one, in which we exclude TEG aid but retain all of the other features of the Carnegie Plan I.

The CED plan we analyze proposes a maximum BEOG of $1,600. For purposes of comparison we have used the Carnegie plan cut-off point of $12,900. Maximum BEOG is paid to students with family incomes of $4,000 or less, and decreases linearly as the family income increases from $4,000 to $12,900. The CED plan would raise public college and university tuitions over ten- and five-year periods respectively until those tuitions were half the cost of instruction per FTE student. Our greatest interest is in the final impact of the plan, so we have treated the targeted final tuition levels as the relevant ones.

Our Equal Opportunity plans require a very flexible, though not unrealistic, aid formula. The first step is to choose the income level which is to be the norm. Students with incomes at or above this norm will receive no aid. Students with family incomes below the norm will receive aid until the ratio of the net cost to their incomes equals the ratio of the cost to the norm income. Net cost is the cost to the norm-income student of attending an IHE minus aid to the lower-than-norm income student.

For example, suppose the norm income was $10,000 and that costs at two types of IHEs were $1,000 and $200 respectively. The student with a family income of $5,000 would be eligible for grants of $500 at the first IHE and $100 at the second IHE. The student with an income of $7,500 would be eligible for a grant of $250 at the more expensive IHE and $50 at the less expensive IHE. For each IHE, the ratios of net cost to income would be the same for all three income groups:

$$\frac{IHE_1 \text{ cost - aid}}{\text{income}} = \frac{\$1000 - 500}{\$5000} = \frac{\$1000 - 250}{\$7500} = \frac{\$1000 - 0}{\$10000} = \frac{1}{10}$$

$$\frac{IHE_2 \text{ cost - aid}}{\text{income}} = \frac{\$200 - 100}{\$5000} = \frac{\$200 - 50}{\$7500} = \frac{\$200 - 0}{\$10000} = \frac{\quad}{50}$$

The aid a student receives clearly depends on how much his family income falls below the norm income and on the cost of the IHE he attends. Aid increases as the difference between his family income and the norm income increases and as the cost of the IHE increases.

We propose two Equal Opportunity plans. Plan I sets the norm income at $14,000 (the evaluation point of the interval $12,900-$15,500) and is comparable to the CED and Carnegie plans in its income cut-off for aid. Plan II is proposed as a test to determine the cost of granting aid to all but the highest income group. The norm income under this plan is $25,000 (the evaluation point for all income greater than $22,000).

In Table 5, we see the baseline freshman demand proportions and the proportions induced by the four plans. In these proportions the ability levels have been averaged by the appropriate population weight for each income level.

TABLE 5

Freshman Enrollment Proportions

A. Baseline

IHE Type \ Income	$6,000	$9,000	$12,000	$14,000	$19,000	$25,000
No IHE	.600	.526	.485	.460	.425	.378
Two-year public	.141	.135	.130	.126	.121	.113
Two-year private	.038	.056	.067	.071	.078	.081
Four-year public	.165	.192	.203	.213	.223	.247
Four-year private	.057	.092	.115	.130	.152	.181

B. Carnegie Plan I

IHE Type \ Income	$6,000	$9,000	$12,000	$14,000	$19,000	$25,000
No IHE	.378	.406	.450	.450	.417	.371
Two-year public	.237	.172	.136	.123	.118	.111
Two-year private	.103	.099	.083	.080	.086	.086
Four-year public	.193	.203	.202	.207	.218	.243
Four-year private	.090	.120	.129	.140	.160	.187

C. Carnegie Plan II (no TEG)

IHE Type \ Income	$6,000	$9,000	$12,000	$14,000	$19,000	$25,000
No IHE	.378	.415	.462	.460	.425	.378
Two-year public	.237	.175	.140	.126	.121	.113
Two-year private	.103	.087	.071	.071	.078	.081
Four-year public	.193	.201	.208	.213	.223	.247
Four-year private	.090	.113	.118	.130	.152	.181

TABLE 5 (continued)

D. CED Plan

IHE Type \ Income	$6,000	$9,000	$12,000	$14,000	$19,000	$25,000
No IHE	.511	.499	.488	.469	.431	.382
Two-year public	.193	.148	.128	.123	.118	.111
Two-year private	.057	.066	.070	.073	.080	.082
Four-year public	.169	.185	.192	.202	.215	.241
Four-year private	.069	.101	.121	.134	.155	.183

E. Equal Opportunity Plan I

IHE Type \ Income	$6,000	$9,000	$12,000	$14,000	$19,000	$25,000
No IHE	.525	.491	.474	.460	.425	.378
Two-year public	.135	.131	.128	.126	.121	.113
Two-year private	.074	.074	.072	.071	.078	.081
Four-year public	.166	.190	.202	.213	.223	.247
Four-year private	.100	.116	.123	.130	.152	.181

F. Equal Opportunity Plan II

IHE Type \ Income	$6,000	$9,000	$12,000	$14,000	$19,000	$25,000
No IHE	.494	.461	.444	.430	.413	.378
Two-year public	.130	.126	.123	.121	.119	.113
Two-year private	.091	.088	.087	.086	.085	.081
Four-year public	.163	.187	.199	.209	.221	.247
Four-year private	.121	.138	.147	.154	.162	.181

Several interesting points can be made about the freshman enrollment demand proportions. Carnegie Plan I increases the proportion of those attending from every income group compared to the baseline. The greatest increase in enrollment comes from the lowest income group. In fact, the proportion not attending any IHE rises from the lowest through the middle income groups. The greatest beneficiary of the increased enrollment are the two-year IHEs, especially the private ones. For the upper income groups, who receive no aid other than TEG, the plan has a predictably small effect; it creates only a slight shift from public to private IHEs.

Carnegie Plan II (no TEG) does not affect the upper income groups at all compared with the baseline, since they receive no aid. Because it does not support private IHE attendance, plan II allows the public IHEs relatively more of the large overall increase in enrollment from lower income groups.

The CED plan increases enrollments among the lower income groups and decreases enrollments slightly among the higher groups. The lowest income group shows increases in enrollments for all IHE types, but even in the two middle-low groups the public four-year IHEs experience a decline. This is clearly the result of decreasing the public-private tuition gap by allowing public IHE tuitions to rise. The same public-to-private shift in enrollments is observed in the upper income groups.

Equal Opportunity Plan I, of course, does not affect enrollments in the upper income groups since there is no change in costs there. In

the lower income groups there is a general increase roughly equivalent to that induced by the CED plan. However, the Equal Opportunity plan shifts enrollments from public to private IHEs with much more frequency than do the other plans, because private IHEs cost more to attend and under this plan aid depends on the cost of attendance.

The effects of the cost-dependent aid in the Equal Opportunity plans are seen clearly by comparing Table 5, parts E and F. Enrollments for all but the highest (the norm) income group have shifted from less expensive public IHEs to more expensive private IHEs. The conclusion to be drawn is that, with this aid formula, as the norm income is increased more expensive IHEs will receive nearly all of the new enrollments as well as some enrollments that formerly went to less expensive IHEs. It is interesting to notice that for the two lowest income groups, even when aid would cover from about 50 to 60 percent of the attendance costs for all types of IHEs, Equal Opportunity Plan II cannot match the Carnegie plans' enrollments for these low income groups. (Compare Table 5, parts A and B, with part F.)

In considering the results from Table 5 (especially those of the last two plans, in view of their names), one might look at our goal of achieving equality of opportunity. The enrollment demand proportions for each income group show that all of the policy proposals are a step in the right direction. But in none of the plans do we see equal demand across income groups save perhaps in the Carnegie plans, where the middle income groups experience the highest proportion not attending any IHE.

Recall that our definition of equality of opportunity states that for a given academic ability level the demand proportions should not differ by income. In Table 6 we show the enrollment demand proportions of different income groups for each ability level under the various plans.

We see the effect of income on enrollment demand in the baseline proportions. At each ability level enrollments increase with income. It is interesting to observe the shift from public four-year IHEs to private four-year IHEs as income rises within the higher two ability groups.

Table 6 shows clearly the reason for the very large increase in lower-income enrollment demand proportions induced by the Carnegie plans. At every ability level overall demand decreases as income rises to the middle range. Of course, demand is not less than the baseline proportions. Lower-ability, lower-income demand is weighted heavily toward two-year IHEs since lower-ability students are not generally eligible for the more highly selective four-year IHEs. Also we note the modest contribution of the TEG proposal in shifting demand from public to private IHEs. All in all, the Carnegie plans are admirable in creating a large increase in demand, but in terms of our definition of equality of opportunity they have achieved this by "reverse discrimination" against the middle income groups.

The CED plan does remarkably well by our equality of opportunity criterion. Overall, enrollment, although not constant for given ability, is fairly stable as income rises. The shift in enrollments from public

TABLE 6

Demand Proportions For Ability and Income Groups

Enrollment Demand Proportions By Ability Groups*

A. Baseline

Number 1 Ability Group

Income	No IHE	2-yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.828	0.144	0.028		
9,000	0.799	0.151	0.050		
12,000	0.780	0.154	0.066	--Not eligible to attend--	
14,000	0.771	0.154	0.075		
19,000	0.755	0.155	0.090		
25,000	0.742	0.156	0.102		

Number 2 Ability Group

Income	No IHE	2-yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.471	0.157	0.051	0.246	0.075
9,000	0.411	0.145	0.068	0.255	0.121
12,000	0.378	0.138	0.077	0.257	0.151
14,000	0.036	0.134	0.081	0.256	0.165
19,000	0.340	0.128	0.088	0.255	0.190
25,000	0.324	0.123	0.092	0.254	0.207

Number 3 Ability Group

Income	No IHE	2-yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.478	0.157	0.052	0.238	0.074
9,000	0.419	0.145	0.069	0.248	0.119
12,000	0.386	0.138	0.078	0.249	0.148
14,000	0.372	0.134	0.082	0.249	0.162
19,000	0.349	0.128	0.089	0.249	0.186
25,000	0.333	0.124	0.093	0.247	0.203

Number 4 Ability Group

Income	No IHE	2-yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.062	0.048	0.029	0.663	0.198
9,000	0.055	0.043	0.031	0.645	0.226
12,000	0.051	0.041	0.032	0.635	0.241
14,000	0.050	0.040	0.032	0.630	0.248
19,000	0.048	0.039	0.032	0.623	0.259
25,000	0.046	0.038	0.033	0.617	0.266

Number 5 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.038	0.032	0.019	0.501	0.410
9,000	0.033	0.028	0.019	0.469	0.451
12,000	0.030	0.026	0.020	0.453	0.472
14,000	0.029	0.025	0.020	0.446	0.481
19,000	0.027	0.024	0.020	0.434	0.495
25,000	0.026	0.023	0.020	0.427	0.505

*Note: Ability is increasing from Group 1 to Group 5.

TABLE 6 (Continued)

Enrollment Demand Proportions By Ability Groups*

B. Carnegie Plan I (with TEG)

Number 1 Ability Group

Income	No IHE	2-yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.560	0.319	0.121		
9,000	0.651	0.230	0.119		
12,000	0.738	0.170	0.092	--Not eligible to attend--	
14,000	0.759	0.152	0.089		
19,000	0.744	0.153	0.102		
25,000	0.734	0.154	0.112		

Number 2 Ability Group

Income	No IHE	2-yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.253	0.191	0.098	0.314	0.143
9,000	0.287	0.156	0.099	0.284	0.175
12,000	0.340	0.138	0.090	0.257	0.176
14,000	0.351	0.130	0.090	0.248	0.182
19,000	0.330	0.124	0.094	0.248	0.203
25,000	0.316	0.121	0.097	0.248	0.218

Number 3 Ability Group

Income	No IHE	2-yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.262	0.191	0.099	0.306	0.142
9,000	0.296	0.156	0.100	0.277	0.172
12,000	0.348	0.138	0.091	0.250	0.172
14,000	0.360	0.130	0.090	0.242	0.178
19,000	0.339	0.125	0.095	0.242	0.199
25,000	0.325	0.122	0.098	0.242	0.214

Number 4 Ability Group

Income	No IHE	2-yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.041	0.045	0.033	0.655	0.225
9,000	0.044	0.041	0.033	0.636	0.245
12,000	0.048	0.040	0.033	0.625	0.253
14,000	0.049	0.040	0.033	0.620	0.258
19,000	0.047	0.038	0.033	0.615	0.266
25,000	0.046	0.037	0.033	0.612	0.272

Number 5 Ability Group

Income	No IHE	2-yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.025	0.029	0.021	0.477	0.449
9,000	0.025	0.026	0.021	0.452	0.477
12,000	0.028	0.025	0.020	0.439	0.488
14,000	0.028	0.024	0.020	0.434	0.494
19,000	0.027	0.023	0.020	0.425	0.505
25,000	0.026	0.022	0.020	0.420	0.512

*Note: Ability is increased from Group 1 to Group 5.

TABLE 6 (Continued)

Enrollment Demand Proportions By Ability Groups*

C. Carnegie Plan II (no TEG)

Number 1 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.560	0.319	0.121		
9,000	0.663	0.234	0.103		
12,000	0.752	0.173	0.075	--Not eligible to attend--	
14,000	0.771	0.154	0.075		
19,000	0.755	0.155	0.090		
25,000	0.742	0.156	0.102		

Number 2 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.253	0.191	0.098	0.314	0.143
9,000	0.295	0.160	0.091	0.292	0.161
12,000	0.353	0.143	0.080	0.267	0.157
14,000	0.363	0.134	0.081	0.256	0.165
19,000	0.340	0.128	0.088	0.255	0.190
25,000	0.324	0.123	0.092	0.254	0.207

Number 3 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.262	0.191	0.099	0.306	0.142
9,000	0.304	0.161	0.092	0.284	0.159
12,000	0.362	0.144	0.081	0.259	0.154
14,000	0.373	0.134	0.082	0.249	0.162
19,000	0.349	0.128	0.089	0.249	0.186
25,000	0.333	0.124	0.093	0.247	0.203

Number 4 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.041	0.045	0.033	0.655	0.225
9,000	0.044	0.042	0.032	0.644	0.237
12,000	0.049	0.041	0.032	0.636	0.242
14,000	0.050	0.040	0.032	0.630	0.248
19,000	0.048	0.039	0.032	0.623	0.259
25,000	0.046	0.038	0.033	0.617	0.266

Number 5 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.025	0.029	0.021	0.477	0.499
9,000	0.026	0.026	0.020	0.461	0.466
12,000	0.029	0.026	0.020	0.453	0.473
14,000	0.029	0.025	0.020	0.446	0.481
19,000	0.027	0.024	0.020	0.434	0.495
25,000	0.026	0.023	0.020	0.427	0.505

*Note: Ability is increasing from Group 1 to Group 5.

TABLE 6 (Continued)

Enrollment Demand Proportions By Ability Groups*

D. CED Plan

Number 1 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.723	0.226	0.051		
9,000	0.764	0.172	0.063		
12,000	0.781	0.149	0.070	--Not eligible to attend--	
14,000	0.778	0.146	0.076		
19,000	0.760	0.149	0.091		
25,000	0.747	0.151	0.103		

Number 2 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.382	0.190	0.069	0.258	0.101
9,000	0.385	0.154	0.077	0.246	0.137
12,000	0.384	0.137	0.081	0.239	0.158
14,000	0.375	0.132	0.084	0.239	0.170
19,000	0.348	0.127	0.090	0.242	0.194
25,000	0.329	0.122	0.093	0.244	0.211

Number 3 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.390	0.189	0.070	0.251	0.100
9,000	0.394	0.154	0.078	0.240	0.135
12,000	0.392	0.138	0.082	0.233	0.155
14,000	0.383	0.133	0.085	0.233	0.167
19,000	0.356	0.127	0.091	0.236	0.190
25,000	0.338	0.123	0.094	0.238	0.207

Number 4 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.055	0.050	0.032	0.647	0.216
9,000	0.053	0.044	0.033	0.631	0.239
12,000	0.052	0.042	0.033	0.623	0.250
14,000	0.051	0.041	0.033	0.619	0.256
19,000	0.049	0.039	0.033	0.615	0.264
25,000	0.047	0.038	0.033	0.611	0.270

Number 5 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.033	0.032	0.020	0.477	0.437
9,000	0.031	0.028	0.020	0.452	0.469
12,000	0.030	0.026	0.020	0.439	0.484
14,000	0.030	0.025	0.020	0.434	0.491
19,000	0.028	0.024	0.020	0.426	0.503
25,000	0.027	0.023	0.020	0.420	0.510

*Note: Ability is increasing from Group 1 to Group 5.

TABLE 6 (Continued)

Enrollment Demand Proportions By Ability Groups*

E. Equal Opportunity Plan I

Number 1 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.771	0.154	0.075		
9,000	0.771	0.154	0.075		
12,000	0.771	0.154	0.075	--Not eligible to attend--	
14,000	0.771	0.154	0.075		
19,000	0.755	0.155	0.090		
25,000	0.742	0.156	0.102		

Number 2 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.363	0.134	0.081	0.256	0.165
9,000	0.363	0.134	0.081	0.256	0.165
12,000	0.363	0.134	0.081	0.256	0.165
14,000	0.363	0.134	0.081	0.256	0.165
19,000	0.340	0.128	0.088	0.255	0.190
25,000	0.324	0.123	0.092	0.254	0.207

Number 3 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.372	0.134	0.082	0.249	0.162
9,000	0.372	0.134	0.082	0.249	0.162
12,000	0.372	0.134	0.082	0.249	0.162
14,000	0.372	0.134	0.082	0.249	0.162
19,000	0.349	0.128	0.089	0.249	0.186
25,000	0.333	0.124	0.093	0.247	0.203

Number 4 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.050	0.040	0.032	0.630	0.248
9,000	0.050	0.040	0.032	0.630	0.248
12,000	0.050	0.040	0.032	0.630	0.248
14,000	0.050	0.040	0.032	0.630	0.248
19,000	0.048	0.039	0.032	0.623	0.259
25,000	0.046	0.038	0.033	0.617	0.266

Number 5 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.029	0.025	0.020	0.446	0.481
9,000	0.029	0.025	0.020	0.446	0.481
12,000	0.029	0.025	0.020	0.446	0.481
14,000	0.029	0.025	0.020	0.446	0.481
19,000	0.027	0.024	0.020	0.434	0.495
25,000	0.026	0.023	0.020	0.427	0.505

*Note: Ability is increasing from Group 1 to Group 5.

TABLE 6 (Continued)

Enrollment Demand Proportions By Ability Groups*

F. Equal Opportunity Plan II

Number 1 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.742	0.156	0.102		
9,000	0.742	0.156	0.102		
12,000	0.742	0.156	0.102	--Not eligible to attend--	
14,000	0.742	0.156	0.102		
19,000	0.742	0.156	0.102		
25,000	0.742	0.156	0.102		

Number 2 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.324	0.123	0.092	0.254	0.207
9,000	0.324	0.123	0.092	0.254	0.207
12,000	0.324	0.123	0.092	0.254	0.207
14,000	0.324	0.123	0.092	0.254	0.207
19,000	0.324	0.123	0.092	0.254	0.207
25,000	0.324	0.123	0.092	0.254	0.207

Number 3 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.333	0.124	0.093	0.247	0.203
9,000	0.333	0.124	0.093	0.247	0.203
12,000	0.333	0.124	0.093	0.247	0.203
14,000	0.333	0.124	0.093	0.247	0.203
19,000	0.333	0.124	0.093	0.247	0.203
25,000	0.333	0.124	0.093	0.247	0.203

Number 4 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.046	0.038	0.033	0.617	0.266
9,000	0.046	0.038	0.033	0.617	0.266
12,000	0.046	0.038	0.033	0.617	0.266
14,000	0.046	0.038	0.033	0.617	0.266
19,000	0.046	0.038	0.033	0.617	0.266
25,000	0.046	0.038	0.033	0.617	0.266

Number 5 Ability Group

Income	No IHE	2-Yr. Public	2-Yr. Private	4-Yr. Public	4-Yr. Private
$ 6,000	0.026	0.023	0.020	0.427	0.505
9,000	0.026	0.023	0.020	0.427	0.505
12,000	0.026	0.023	0.020	0.427	0.505
14,000	0.026	0.023	0.020	0.427	0.505
19,000	0.026	0.023	0.020	0.427	0.505
25,000	0.026	0.023	0.020	0.427	0.505

*Note: Ability is increasing from Group 1 to Group 5.

to private institutions occurs mostly at the expense of the two-year IHEs and to the benefit of the four-year private IHEs.

Equal Opportunity Plans I and II in fact equalize opportunity for income groups of up to $14,000 and $25,000 respectively. The interesting point is that the CED method of closing the tuition gap achieves nearly the same result as does ours. We effectively reduce the absolute difference between public and private IHE costs by reducing costs proportionately.

In Table 7 we show the increases from the baseline in FTE enrollments, first-time enrollments, and degrees induced by each of the plans. The contrast between the nearly equal increase in two-year IHE and four-year IHE enrollments under the CED plan and the large increase in four-year IHE enrollment under the Equal Opportunity plans is apparent. In this respect, the Carnegie plans are similar to the CED plan. All of the plans induce substantial increases in enrollments.

TABLE 7

Increases above Baseline Enrollment and Degree Projections (in thousands)

A. Carnegie Plan I

Year	Total FTE	2-Yr. FTE	4-Yr. FTE	A.A.	B.A.	First-time 2-Yr.	First-time 4-Yr.
1972	735	297	436	36	93	144	63
1973	747	303	447	39	96	146	66
1974	763	305	443	38	100	148	65
1975	775	311	462	40	103	149	66
1976	781	312	469	40	103	150	67
1977	788	315	473	41	106	151	67
1978	792	314	478	40	108	149	66
1979	792	311	480	41	108	148	66
1980	785	307	477	41	108	146	64
1981	778	304	475	40	110	143	63
1982	764	297	467	40	109	140	61
1983	749	287	459	39	110	132	59

TABLE 7 (Continued)

B. Carnegie Plan II (no TEG)

Year	Total FTE	2-Yr. FTE	4-Yr. FTE	A.A.	B.A.	First-time 2-Yr.	First-time 4-Yr.
1972	671	267	403	32	86	129	60
1973	683	271	412	35	90	130	62
1974	696	274	420	35	92	133	61
1975	706	279	427	36	94	133	62
1976	714	281	431	35	95	134	63
1977	719	283	438	37	98	135	63
1978	723	282	440	36	99	134	62
1979	722	279	442	37	99	132	62
1980	716	275	441	37	100	130	60
1981	709	273	437	37	102	128	60
1982	698	266	432	35	102	125	58
1983	681	257	424	36	100	119	55

TABLE 7 (Continued)

C. CED Plan

Year	Total FTE	2-Yr. FTE	4-Yr. FTE	A.A.	B.A.	First-time 2-Yr.	First-time 4-Yr.
1972	209	105	104	12	22	50	9
1973	211	106	106	13	25	52	9
1974	215	107	108	13	25	52	9
1975	219	110	111	13	26	52	8
1976	221	110	110	14	25	53	9
1977	223	109	112	14	28	53	9
1978	224	109	113	14	28	53	10
1979	223	109	114	14	27	52	9
1980	221	107	114	14	27	52	8
1981	219	107	114	14	28	50	9
1982	216	103	113	14	29	49	9
1983	212	100	110	14	27	46	8

TABLE 7 (Continued)

D. Equal Opportunity Plan I

Year	Total FTE	2-Yr. FTE	4-Yr. FTE	A.A.	B.A.	First-time 2-Yr.	First-time 4-Yr.
1972	257	43	214	6	41	19	50
1973	262	43	218	6	44	19	52
1974	265	42	222	7	44	20	51
1975	270	44	226	6	45	19	51
1976	273	44	228	6	46	21	53
1977	276	45	230	6	47	20	53
1978	276	44	231	6	48	20	52
1979	276	44	232	6	48	20	52
1980	273	44	231	6	48	19	51
1981	271	43	227	7	49	19	50
1982	267	42	224	6	48	19	48
1983	260	41	220	6	47	17	46

TABLE 7 (Continued)

E. Equal Opportunity Plan II

Year	Total FTE	2-Yr. FTE	4-Yr FTE	A.A.	B.A.	First-time 2-Yr.	First-time 4-Yr.
1972	377	74	302	9	57	33	69
1973	382	75	310	9	62	35	70
1974	390	75	313	10	63	36	71
1975	397	78	320	10	64	35	71
1976	400	77	323	10	64	36	72
1977	405	78	326	10	67	36	72
1978	406	77	328	10	68	36	72
1979	405	77	328	11	66	35	72
1980	401	75	326	11	68	35	69
1981	397	74	323	11	69	34	68
1982	392	74	318	10	69	33	66
1983	382	71	312	10	68	31	64

IV. Use of the Model: The Costs of the Plans

In calculating the costs of the five plans, we have continued to assume that the baseline has zero cost, that each plan has been fully funded, and that each plan began by at least 1967 so that by 1972 every student in higher education had made his enrollment decision under the costs adjusted by the plans. We also have assumed that the fraction of public to private enrollments in the transition model does not vary from those of the freshman demand model, and that the proportion of average cost of institution covered by tuition is one-quarter for public IHEs and four-fifths for private IHEs. Under these assumptions we have calculated the cost of each plan per student in each income group attending public and private IHEs.

Table 8 shows the direct aid costs per student for the three lowest income groups of Carnegie Plan I (with TEG). The only cost in higher income groups is $750 (TEG), when the IHE chosen is privately controlled.

TABLE 8

Carnegie Plan I (with TEG) Average Aid Cost per Student by Income Level and IHE Choice (in 1974 dollars)

Income Level less than $7,500

	BEOG	SSIG	TEG	Total Aid Cost
Public two-year	$1,600	$406	$ 0	$2,006
Private two-year	1,600	750	750	3,100
Public four-year	1,600	593	0	2,193
Private four-year	1,600	750	750	3,100

Income Level between $7,500 and $10,500

	BEOG	SSIG	TEG	Total Aid Cost
Public two-year	$1,177	$406	$ 0	$1,583
Private two-year	1,177	750	750	2,677
Public four-year	1,177	593	0	1,773
Private four-year	1,177	750	750	2,677

Income Level between $10,500 and $12,900

	BEOG	SSIG	TEG	Total Aid Cost
Public two-year	$272	$255	$ 0	$ 527
Private two-year	272	255	750	1,277
Public four-year	272	255	0	527
Private four-year	272	255	750	1,277

The costs of Carnegie Plan II (no TEG) are very similar to those of Carnegie Plan II (with TEG). The BEOG costs per student are the same in both plans. Table 9 shows the SSIG cost per student.

TABLE 9

Carnegie Plan II--SSIG Cost per Student

IHE Type / Income	$6,000	$9,000	$12,000
2-year public	$ 406	$ 406	$255
2-year private	1,500	1,109	255
4-year public	593	593	255
4-year private	1,500	1,109	255

The CED and Equal Opportunity plans provide for only one form of aid. CED plan aid is contingent on income alone; the lowest-income-group student receives $1,240; the $9,000-income group student $701; and the highest-income-group ($12,000) student eligible for aid receives $240. Equal Opportunity Plan I average grants per student are shown in Table 10.

TABLE 10

Equal Opportunity Plan I--Average Grant per Student

IHE Type / Income	$6,000	$9,000	$12,000
2-year public	$ 232	$ 145	$ 58
2-year private	1,760	1,100	440
4-year public	800	540	202
4-year private	2,560	1,600	640

Finally, we have the average grants per student under the Equal Opportunity Plan II in Table 11.

TABLE 11

Equal Opportunity Plan II--Average Grant per Student

IHE Type \ Income	$6,000	$9,000	$12,000	$14,000	$19,000	$25,000
2-year public	309	260	211	179	97	0
2-year private	2,341	1,971	1,602	1,355	739	0
4-year public	1,079	916	747	634	347	0
4-year private	3,405	2,867	2,330	1,971	1,075	0

The range of aid per student under the various plans is remarkable. The $6,000-income student receives $1,240 under the CED plan regardless of which type of IHE he chooses. The Carnegie plans give him up to $3,100 or as little as $2,006 depending on the institution he attends. Equal Opportunity Plan I gives him a (comparatively) meager $232 to defray the costs of a low-cost, public, two-year college, and as much as $2,560 to go to an expensive, four-year, private university. The corresponding choices to the same student under the Equal Opportunity Plan II would yield $309 and $3,405 respectively.

In Table 12, Part 1, we show the total aid costs of the Carnegie Plan I (with TEG) by aid-type and income level.

TABLE 12

Part 1. Total Aid Cost (1976) of Carnegie Plan I (with TEG) (in millions of 1974 dollars)

Income	BEOG	SSIG	TEG	Total for Income
$ 6,000	$2,624	$935	$380	$3,939
9,000	1,238	629	291	2,158
12,000	330	215	244	689
14,000	0	0	244	244
19,000	0	0	351	351
25,000	0	0	554	554
Total Program	$4,092	$1,779	$2,064	$7,935

The largest aid expenditures go, of course, to the lowest income group. BEOG is the most expensive of all the forms of aid.

In Table 12, Part 2, we show the cost of Carnegie Plan II (no TEG). Other plans are constrained by the requirement that TEG plus SSIG must be less than $1,500. Since in this case TEG is zero, which means that SSIG must be less than $1,500 the SSIG cost has increased.

TABLE 12 (Continued)

Part 2. Aid Cost of Carnegie Plan II (in millions of 1974 dollars)

Income	BEOG	SSIG	Total for Income
$ 6,000	$2,600	$1,347	$3,874
9,000	1,218	770	1,994
12,000	224	208	425
Total Program	$4,032	$2,233	$6,313

The CED plan by income-group costs are shown in Table 12, Part 3.

TABLE 12 (Continued)

Part 3. Aid Cost of the CED Plan (in millions of 1974 dollars)

Income	Cost
$ 6,000	$1,655
9,000	637
12,000	183
Total	$2,475

The last parts of Table 12 show the projected cost of the Equal Opportunity plans, again for 1976 in 1974 dollars.

TABLE 12 (Continued)

Part 4. Total Aid Cost of Equal Opportunity Plan I

Income	Cost
$ 6,000	$1,840
9,000	764
12,000	278
Total	$2,882

Table 12 (Continued)

Part 5. Total Aid Cost of Equal Opportunity Plan II

Income	Cost
$ 6,000	$2,582
9,000	1,534
12,000	1,090
14,000	896
19,000	631
25,000	0
Total	$6,733

In terms of those costs, the plans fall into two groups according to expense. The CED plan and the Equal Opportunity Plan I are far less expensive than the Carnegie plans or Equal Opportunity Plan II.

Aid paid to students is not the only cost incurred when enrollments in IHEs grow. Another expense is the cost to the IHE, or to its provider of funds, that results from tuition being less than the average cost of instruction. We call this induced cost.

TABLE 13

Induced Costs of the Plans
(in millions of 1974 dollars)

Carnegie Plan I	$1,023
Carnegie Plan II	942
CED Plan*	221
Equal Opportunity Plan I	304
Equal Opportunity Plan II	518

*One might infer that the induced cost of the CED plan would be negative since all students at public IHEs would be paying an increased share of the average cost of instruction. However, this is simply a transfer of the cost from the taxpaying public to student family public.

Table 13 shows that again the Carnegie plans are the most costly. This is in the main due to the great increase in enrollments generated by the plans, and to a lesser extent to the relatively greater increase in public IHE enrollments; the induced cost per student is higher for public IHEs than it is for private IHEs.

The CED plan is still the least expensive and has the smallest

relative induced cost with respect to aid cost. This is due to the decrease in the public IHE subsidy.

TABLE 14

Total Costs of the Plans
(in millions of 1974 dollars)

	Aid Cost	Induced Cost	Total Cost
Carnegie Plan I	$7,935	$1,023	$8,958
Carnegie Plan II	6,313	942	7,255
CED Plan	2,475	221	2,696
Equal Opportunity Plan I	2,882	304	3,186
Equal Opportunity Plan II	6,733	518	7,251

In comparing both types of costs and totals in Table 14 it is important to note that, although the aid costs dominate the induced costs, the latter are not proportional to the former. The type of new enrollments generated by a policy plan will influence the induced cost differently than it will influence aid cost. Relatively low-aid public enrollments can imply relatively high induced costs.

The more expensive plans "buy" more enrollments. The Carnegie plans induce the greatest increase in enrollments by considerably expanding lower aid-cost enrollments. Equal Opportunity Plan II, although nearly as expensive, encourages a much smaller total enrollment increase due to the shift to expensive IHEs.

The less expensive plans differ little from each other except in that Equal Opportunity Plan I tends to shift enrollments more to expensive

IHEs than does the CED plan. The CED plan is the least expensive and in a sense the most "progressive" in that it provides a larger proportion of aid for the lowest income group than do the other plans.

The plans can also be compared according to how much aid each spends to induce one new enrollment.[6] Care must be used in this comparison because, to our minds at least, a new two-year IHE enrollment is not qualitatively equal to a four-year IHE enrollment. Bearing this in mind, in Table 15 we see that this comparison considerably changes the rankings, by cost, of the plans.

TABLE 15

Cost per New Enrollment

	Change in Enrollments (thousands)	Aid (millions of dollars)	Aid per New Enrollment (dollars)
Carnegie Plan I	781	$7,935	$10,161
Carnegie Plan II	714	6,313	8,842
CED Plan	221	2,475	11,199
Equal Opportunity Plan I	273	2,882	10,557
Equal Opportunity Plan II	400	6,733	16,832

In the sense that they produce the highest number of enrollments per dollar, the Carnegie plans are the most efficient and Equal Opportunity Plan II is in an expense class by itself. These characteristics can be explained by the fact that in our model the price elasticity of demand is greatest for the lowest income group and decreases as income

[6]This comparison was suggested by George Weathersby.

rises. The Carnegie plans provide much larger average grants to the lowest-income students than do the other plans. For these plans the price elasticity and income relationship implies more new enrollments per aid dollar, or, looking at the same thing from the other side, a lower dollar aid cost per new enrollment.

Equal Opportunity Plan I is something of a compromise between the CED Plan and Carnegie Plan I. While granting less aid than Carnegie Plan I, it does not increase tuition as does the CED plan. The increase in tuition proposed by the CED plan results in a small loss of enrollments for the upper income groups and increases the cost per new enrollment. Equal Opportunity Plan II fares poorly by this comparison. This is due to its granting aid to every student, old or new, except those in the highest income group.

One point that is immediately clear from Table 15 is that in no case is the aid cost of inducing new enrollments small. A considerable amount of aid is paid to students attending any IHE. Of course, the inability to efficiently target aid is not unique to aid plans in higher education.

One plausible alternative is a plan granting aid by the Equal Opportunity plan formula that also allows public tuitions to rise as in the CED plan. The expected shift away from public schools to either no school or to private schools would be lessened because the students with incomes below the norm would bear only part of the tuition increase. (This could be one reason for public schools to raise their tuitions in

the first place). Our rough calculations indicate that if public tuitions were to rise to cover half of the cost of instruction (with norm income at $14,000), the aid cost of Equal Opportunity Plan I would increase to about $4 billion.

V. Critique of the National Commission on the Financing of Postsecondary Education Model

The National Commission on the Financing of Postsecondary Education has developed a model of many aspects of the education industry.[7] In particular, this model forecasts student enrollments in institutions of higher education given net costs of attendance--also forecasts the funding those institutions would need to provide the educational services demanded. We shall present a critique of a subset of the whole National Commission on the Financing of Postsecondary Education (NCFPE) model, namely, that part describing student enrollments. Henceforth the term NCFPE model will refer to this subset.

The NCFPE Model

We can write the NCFPE model in a very simple form:[8]

$$BE^* = BE\left(1 + \frac{\frac{\partial P}{\partial C}}{P}\right) \qquad (1)$$

where

P = proportion of high school enrollment (HSE) attending postsecondary education.

$\frac{\partial P}{\partial C}$ = the change in the proportion P per unit change in the cost of attendance C.

BE = baseline postsecondary enrollments.

BE^* = postsecondary enrollments after a change in the cost of attendance.

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[7] The model is presented and discussed in A Framework for Analyzing Postsecondary Education Policies: A Staff Report; Carlson, Farmer, Weathersby, May 1974. United States Government Printing Office. We refer to this publication as the Staff Report.

[8] See the Staff Report, p. 54.

In other words, predicted enrollments are the sum of two components. The first is the estimated trend enrollments. The second is the deviation from the trend due to a change in the cost of attendance. BE is called the baseline enrollment and $\left[\frac{\frac{\partial P}{\partial C}}{P}\right]$ is the price-response coefficient. The change in the cost of attendance is the policy instrument.

In order to discriminate between enrollments in different types of IHEs by students of different income levels given the policy makers' schedule of cost changes, the NCFPE uses nine types of IHEs and eleven students' income levels shown in Table 13. The model can now be written:[9]

$$BE^*_{ij} = BE_{ij}\left(1 + \sum_{k=1}^{9} PRC_{ijk}\Delta C_k\right) \qquad (2)$$

where

ΔC_k = the change in the cost of attendance of the k^{th} type IHE.

PRC_{ijk} = the change in the proportion of the i^{th} income level enrolling in the j^{th} IHE due to a change in the cost of attendance to the k^{th} IHE.

BE_{ij} = baseline enrollments to the j^{th} IHE by the i^{th} income level students.

BE^*_{ij} = predicted enrollments of i^{th} income level students in the j^{th} IHE given changes in the cost of attendance of all IHEs.

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[9] See the Staff Report, p. 66.

TABLE 13

NCFPE IHE Types

1. Public two-year
2. Public four-year, lower division
3. Public four-year, upper division
4. Public four-year, graduate
5. Private two-year
6. Private four-year, lower division
7. Private four-year, upper division
8. Private four-year, graduate
9. Noncollegiate

NCFPE Family Income Categories and Midpoints

1	$ 0 - 999	$ 500
2	1,000 - 1,999	1,500
3	2,000 - 2,999	2,500
4	3,000 - 3,999	3,500
5	4,000 - 4,999	4,500
6	5,000 - 5,999	5,500
7	6,000 - 7,499	6,750
8	7,500 - 9,999	8,750
9	10,000 - 14,999	12,500
10	15,000 - 24,999	20,000
11	25,000 - all above	50,000

The NCFPE derives its baseline enrollment projections from two sources. The first is the National Center for Educational Statistics, which has projected total enrollments for each year from 1974 to 1985. The NCFPE apportions each year's projected enrollments to the nine IHE types. The second is the United States Bureau of the Census Special Tabulations (October, 1972), which gives the distribution of enrollments of each of the income levels across IHE types. The NCFPE combines the 1972 (published) income level and IHE type enrollment distribution with

the projected IHE enrollments to give baseline enrollments of income level to IHE type.

The baseline IHE cost data have been derived from the United States Office of Education, HEGIS (1971-1972). The change in the cost of attendance to an IHE is the new policy makers' cost minus the baseline cost.

The price-response coefficients (PRC) are the behavioral heart of the model. They represent the percentage change from baseline attendance for an IHE type given a change in cost in attendance for students of specified income levels. The NCFPE uses a $100 cost change to calculate the PRCs, which they derive from Radner's and Miller's estimated model and data set.[10] The derivation is not simple, because the Radner-Miller model recognizes differences in student academic ability and IHE selectivity and uses a different IHE categorization. Considerations of IHE category differences aside, the NCFPE price-response coefficients have the following relationship with the Radner-Miller model:

1) The NCFPE family income categories 1 through 7 (income less than $7,500), called low income, have the same price responses. The low-income PRC represents the price response of a student of average ability with a family income of less than $7,500 from the Radner-Miller sample set and model.

2) The NCFPE family income categories 8 and 9 (income greater than or equal to $7,500 but less than $15,000), called middle income, have the same price responses. The middle-income PRC represents the price responses of a student of average ability within that income range from the Radner-Miller sample set and model.

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[10] The Staff Report, Appendix C.

3) The NCFPE family income categories 10 and 11, called high income (income greater than or equal to $15,000), have the same price responses. The high-income PRC are the price responses of the average-ability, high-income students from the Radner-Miller sample set and model.

In all cases the Radner-Miller data used for the derivation of the PRCs assume that the student lives within commuting distance of all (Radner-Miller) IHE types.

The Radner-Miller model estimates freshman demand for postsecondary education: the NCFPE model predicts all students' demands for post-secondary education. To obtain a correspondence between the different IHE categorizations the NCFPE assumes the following:[11]

1) All students in two-year or noncollegiate IHEs have the same price responses as do the freshmen in comparable Radner-Miller categories.

2) All students in public four-year IHEs have the same price responses. These price responses are the average freshmen responses in the four Radner-Miller public, four-year IHE categories.

3) All students in private, four-year IHEs have the same price responses. These price responses are the average of freshmen responses in the three Radner-Miller private, four-year IHE categories.

4) The indirect price response between two IHEs is zero when the IHEs cannot be substituted: for example, a change in the cost of going to a public four-year, upper-division IHE does not change enrollments in lower-division or graduate IHEs.

Improving the NCFPE Model

We believe that the NCFPE model would be improved by adding the dimensions of student academic ability and IHE selectivity based on

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[11] Appendix II shows the details of the IHE mapping. See also Appendix C of the Staff Report.

academic ability. With this specification a student is described by family income and by a measure of academic ability. In addition, IHEs are described by their cost of attendance and by a measure of how academically selective they are. This specification is useful for the following reasons.

1) Previous research has shown that ability is a significant factor in determining the type, when selectivity is specified, of IHE attended.[12]

2) The additional information about who goes where is potentially valuable. Policies aimed at increasing equality of opportunity might be different (and improved, we hope) using this information.

3) Accounting for ability is in accordance with the economic theory of human capital. Academic ability can be considered an indicator of a previous investment in human capital. Alternatively, it might serve as a proxy for future expected returns on investment in education.

4) Accounting for ability is more in the spirit of the source of the input data used to determine the NCFPE price-response coefficients. Since the PRCs are the behavioral part of the NCFPE model, it is better that the assumptions and estimating procedures on which they are based be straightforward and easy to see.

5) A simple use of this specification requires little new data. One can hypothesize the joint distribution of income and ability in the population and the distribution of selectivity among IHEs. However,

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[12] Radner and Miller, op.cit. Kohm, Manski, and Mundel, An Empirical Investigation of Factors Which Influence College Going Behavior, October, 1973.

empirical information as to these distributions is the preferred alternative.

We believe that the NCFPE model would be improved by adding the geographic distribution of persons and IHEs. The NCFPE does not take into account the large increase in the cost of attending an IHE that is not within commuting distance as opposed to one that is. For the typical public IHE, this added cost is greater than the direct costs of attendance.

We note that the NCFPE could have derived price-response coefficients in several equally defensible ways. In a rough sense the NCFPE price-response coefficients represent the price response of an average person within an income group. Using exactly the same data, the price responses can be calculated to represent the average price response of all persons within an income class. Because the PRCs are not linear functions of the parameters[13] these two methods of calculation should yield different price responses.

The baseline calculations are not neutral with respect to policy cost changes. They embody the past trends in policy. In particular the trends in costs of attendance are important because we would like to know if the price changes used to calculate the PRC, or even the policy changes in costs, are different from the trend costs. For large cost changes the model itself should be reevaluated.[14]

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[13] In Appendix II we discuss alternative formulations of the PRC.

[14] This point was recognized by NCFPE, e.g., the Staff Report, p. 75.

A Comparison of our Model with the NCFPE Model

We expected that our analysis would differ from that of the NCFPE in several respects. By omitting academic ability, the NCFPE model cannot be scrutinized from the perspective of our definition of equal opportunity. As we noted in our discussion of the policy plans, equalizing opportunity across income groups requires significantly greater demand by low-income, low-ability students than by low-ability, higher-income students. This factor has not only independent interest but also, if aimed for, indicates that a large proportion of the demand will focus on the public two-year IHEs.

Another difference is an important technical one. We recall that the NCFPE price-response coefficients are not entirely satisfactory when the changes in the cost of attendance are large. Some of the plans we have considered, and some of those which the NCFPE analyze, contain proposals for very substantial cost changes. Their methods approximate movements along an S-shaped curve by following one of three (low-, medium-, or high-income) tangents to the curve. This tends to underestimate the price response of the student who initially has a low attendance frequency. The larger the cost change, the greater, in general, is the error. Conversely, the NCFPE method tends to overestimate the response of those with high attendance proportions. A comparison of NCFPE enrollment estimates with ours has led us to believe that the NCFPE has in fact estimated a smaller response from low-income students (this group has a larger proportion of lower-ability students)

than we have when the cost changes are large.

A very important difference between the two models is that we assume that the elasticity of demand for students already enrolled in an IHE is zero. The NCFPE assumes that enrolled students and prospective students have the same elasticity of demand over a broad range of IHEs. The true situation is probably somewhere between these assumptions. As the matter stands, the two models predict very different reactions to changes in the cost of attendance for previously enrolled students.

Finally, we use our model to predict the baseline enrollments while the NCFPE model only calculates changes from an exogenous baseline. Since our baseline is not exactly the same as that used by the NCFPE, there are inevitable discrepancies when comparisons are made between predicted total enrollments and costs.

VI. A Closer Look at the Model

Our freshman demand model is an adaptation of the model developed by Radner and Miller.[15] We define the model as:

$$(1) \qquad T^{\pi}oi = F(Z, T^{X}ik)\phi_{ik}$$

where

ϕ_{ik} is the joint distribution of income and ability.

$T^{X}ik$ is the number of T^{th} year high school graduates having family income i and ability k.

Z is the estimated parameters, exogenous and policy variables.

F is the fitted functional form of the Radner-Miller model. We intend this to include the choice set restrictions by ability as well as those due to demographic distributions of persons and IHEs.

$T^{\pi}oi$ is a vector with as many elements as types of IHE that we wish to specify.

Any particular element, say the third, is the number of high school graduates who choose IHE-type three in the year in question. We call this vector the initial vector for income group i since for the i^{th} income level it is a weighted average of all ability levels.

We define the transition model as:

$$(2) \qquad T^{\pi}ti = T^{\pi}oi P^{t}$$

where $T^{\pi}oi$ is from (1), and

P is a square matrix of transition probabilities.

$T^{\pi}ti$ is the number of students having family incomes of level i, from high school class T, in "state" j t years from the beginning of the school year T.

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[15] R. Radner and L.S. Miller, Op. cit.

Together the freshman demand model and the transition model comprise our larger model, which we call the enrollment demand model. By adding up the appropriate elements for each income group we have predicted enrollment demand in year T+t for high school class T. Total enrollment demand for a particular year is found by including several high school classes, each in a different temporal state of progress, through the transition model for the year in question.

The problems that all model builders face were especially apparent to us when we formulated our transition model. The transition model specifies the real world only in that there are fewer states defined in the model than there are states to be identified in the world. If one chooses to define many states and thus closely approximate the real world, one creates two types of problem: the number of computations grows rapidly since each state must have its own row and column in the transition matrix, and there must be identifiable transition probabilities between each and every state. We were less bothered by the first problem only because the second problem was a real constraint. Even in our relatively simple model, we were in several instances unable to assign transition probabilities on any firmer grounds than supposition. We had to accept some imprecision in the transition probabilities as the price for defining what we felt were the minimum number of states. Even so, as we note subsequently, there is an unfortunate loss of information from the freshman demand model to the transition model.

Thirteen of our states are mutually exclusive, and the set of all the states is completely inclusive. In Table 1 we define each of the states. One can see that there are four natural groupings of states: (1) State 1 (in the terminology of Markov chains, this is the absorbing state) "collects" those who will never enter or reenter an institution of higher education; (2) States 2 and 3 include all two-year enrollments; (3) States 4 through 8 include all four-year college and university enrollments; (4) States 9 through 13 include all those who may enroll or reenroll, the holding pool, if you will.

We recall that the freshman demand model considers nine different types of IHEs as well as the option of not attending any IHE (see Table 14). In a sense, the groups of states can be considered as types of IHEs. The manner in which the freshman demand proportions are entered into the transition model is straightforward. All freshman demand for IHE types 3, 4, 6, 7, 8, 9, and 10 is added and entered into State 4 of the transition model. Freshman demand for types 2 and 5 is entered into State 2.

The freshman demand model does not distinguish between two types of non-attendance, States 1 and 9; as a result, how to apportion the not enrolling demand between State 1 and State 9 proved to be difficult. We had thought that the age distribution of first-time enrollments would indicate the (sizeable, we thought) fraction to be in State 9. We found, to the contrary, that the freshman demand model quite accurately predicted first-time enrollments even when all type 1 demand was assigned

to State 1. We concluded that the observable distribution of age of first-time enrollments could be replicated, if desired, by withholding through time declining proportions of the high school graduates from the freshman demand model; that is, in the first equation X_T would be replaced by, say, $aX_T + bX_{T-1} + cX_{T-2} + \ldots$ where the coefficients sum to one. In a steady state the total demand proportions would be unchanged from the case where the whole high school class is entered as X_T.

TABLE 14

Freshman Demand IHE Types

Type	Description
1	No attendance
2	Two-year community college
3	Four-year state college in local area
4	State university in local area
5	Private two-year college or technical school
6	Four-year state college not in local area
7	State university away from home
8	Private four-year college of low selectivity
9	Private four-year college or university of moderate selectivity
10	Highly selective private university

Clearly the transition model makes no distinction between public and private IHEs, the levels of selectivity of IHEs, or the costs or changes in costs of attendance. We have this information in the freshman demand model, but it is lost when we compress the IHE types into the transition states. This awkard and unfortunate situation forces us to make some strong assumptions when calculating the cost of a policy proposal. We assume that the relative proportions of public and private demand remain the same through the transition model as those predicted by the freshman demand model for each income group.

In Table 15 we show the matrix of transition probabilities. Our chief source of attrition data was the large and relatively comprehensive study done by A. Astin.[16] We used his results, and others where appropriate, to sketch transition probabilities from the first year to the second and to outline the chain of probabilities from entrance to graduation. We did not "estimate" the transition probabilities in the econcmetric sense. Our basic criterion was that each probability value chosen was reasonable with respect to Astin, and that as a whole the model should give believable results.

Finally, we placed a restriction on the length of time any high school class could spend within the transition model--a period of six years. This restriction implies that a student is allowed at most two departures from educational progress and

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[16] A. Astin, The College Dropout: National Profile, 1972.

still have any chance to obtain a bachelor's degree. However, his chance to obtain an associate degree remains more flexible.

We reiterate that the transition probabilities we use should not be considered precise estimates based on extensive data analysis. We have been limited by an extreme paucity of information relevant to the transition between most of the states we define. The benchmark measures of attrition we use are those of Astin:

1) The percentage of students who reenroll for the second year at the same IHE.
2) The percentage of students who receive a degree, at their same IHE, within four years after their initial enrollment.
3) The percentage of students who either meet the criteria of the above measure or who are still enrolled at the original IHE after four years.
4) The percentage of students who either meet the definition of the preceding measure or who have at some time within the previous four years requested that their transcript be sent to another IHE.

We did not expect any of Astin's measures of persistence to be the same as our transition probabilities. In the first place, his study is limited to four and a half years. As he notes, increasing the length of time to observe degree completion would have increased considerably his measure of degree success.[17] Secondly, only in his fourth measure is Astin able to consider

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[17] Astin, Op. cit., p. 9.

TABLE 15

Matrix of Transition Probabilities*

i \ j	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1.0												
2	.08		.67							.25			
3	.13		.30			.25					.32		
4	.04				.86					.10			
5	.03					.88					.09		
6	.02						.91					.07	
7	.81							.15					.04
8	.91							.08					.01
9	.15	.20		.03					.62				
10	.10		.25		.45					.20			
11	.10					.60					.30		
12	.05						.75					.20	
13	.05							.80					.15

* All blank elements are zero. Each element is read as the probability of going from state i to state j .

remotely the impact of transfers on degree completions and attendance persistence; as a rule, transferring to a different IHE is measured as dropping out. We explicitly consider transfers only when they are from two-year to four-year IHEs. The effects of the other types of transfers increase our transition probabilities relative to Astin's measure of persistence. Finally, our objective is different from that of Astin. We want our transition probabilities to approximate the average outcome (total enrollments and degrees awarded) of a complex process.

In view of the limitations of attrition and persistence measures of Astin (the same limitations apply to other similar studies) we have proceeded to estimate the transition probabilities using known data as starting points. A general principle we have followed in choosing transition probabilities is that the probability of continuing as a student increases as the amount of completed education increases. From our starting estimates and this latter principle, we have <u>forced</u> the transition probabilities so that the known initial conditions lead to accurate total enrollment and degree predictions. The procedure does not yield unique probabilities, as we have many degrees of freedom.

Consider the transition probabilities associated with students in four-year IHEs (States 4, 5, 6, 7, and 8). They predict that nearly 69 percent of the students who enroll as freshmen will enroll as seniors three years later. The graduation rate for four-year students who take no time off is 0.55; within five years after enrollment

it is o.64. For comparison, consider that another study which projects degrees and total enrollments uses a graduation rate of between 0.50 and 0.55.[18] Considering the large difference in the graduation rate, the degree projections of this study are surprisingly similar to our own. This near agreement is due to quite different projections of undergraduate enrollment.

The transition probabilities for the states representing two-year IHEs conform more closely to Astin's starting estimate than do the four-year probabilities. We believe that this reflects the years (four rather than two) on which Astin bases his two-year IHE student estimates. Our rate of transfers between two-year and four-year IHEs is based on a rough estimate that one-third to one-sixth of the entering students transfer.[19]

There is little to support the transition probabilities relating to entering and leaving the "stop-out" states. These probabilities are the first to be adjusted in the forcing procedure since they are based on so little hard data in the first place. It is clear that they imply a much greater possibility of reenrollment among those who have completed two or more years of higher education.

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[18] G. W. Haggstrom, "The Growth of Higher Education in the United States." Unpublished manuscript prepared for the Carnegie Commission on Higher Education. Berkeley, 1971.

[19] ERIC Clearinghouse on Higher Education; The Number Two Access Problem, 1972.

VII. Conclusion

We have used our model to analyze the impact on enrollment of five different policy plans for student aid in United States higher education. We have calculated the cost of each plan for a typical year (1976) had the students taken advantage of the proposed aid. We have not calculated student response to work-study aid or to student loans, although we are quite certain that these two programs also have an effect on enrollment.

We think our approach provides interesting and in some cases new information concerning topical questions such as how to improve the competitive position of high-cost (private) colleges and universities vis-à-vis lower-cost (public) ones, or how well aid policies improve the chances of students from families of less than average incomes to attend colleges and universities with the same frequency as their more fortunate brethren.

There are several criteria by which we can rank the plans. All propose aid by need rather than by merit. Carnegie Plan I would grant substantial aid ($1,600) to students with family incomes of less than $7,600 and a decreasing proportion of that amount as income rises to the no-aid cut off point of $12,900. Instructional costs of up to $1,500 would be covered or partially covered by the same income-aid formula. Regardless of income, $750 would be granted to students attending a private university or college, although the total private subsidy and instructional cost granted could not exceed $1,500.

Carnegie Plan II would grant aid in exactly the same manner and

amounts as would Carnegie Plan I, except that the $750 grant to students attending a private college or university would be eliminated.

The CED plan is similar to Carnegie Plan II in terms of aid, although it makes no provision for aid to cover instructional costs. However, it does propose to increase public college and university tuition to meet half the cost of instruction.

The Equal Opportunity plans we propose are based on granting aid so that the "net burden of cost" (cost adjusted by grants and divided by family income) does not change as income rises to the income norm. Above that income level no aid is proposed. Equal Opportunity Plan I would grant aid by this procedure to students having an income of less than $12,900. Equal Opportunity Plan II would grant aid to students having a family income of less than $22,000.

We estimate the cost in direct aid of the plans in 1976, had they been funded for at least the previous six years, to be: Carnegie Plan I, $7.9 billion; Equal Opportunity Plan II, $6.7 billion; Carnegie Plan II, $6.3 billion; Equal Opportunity Plan I, $2.9 billion; and the CED plan, $2.5 billion. Adding the induced cost of the plans (the subsidy provided by the IHEs as a result of tuition being less than the average cost of instruction), the total costs would be: Carnegie Plan I, $9.0 billion; Carnegie Plan II, $7.3 billion; the CED plan, $2.7 billion; Equal Opportunity Plan I, $3.2 billion; and Equal Opportunity Plan II, $7.3 billion. All amounts are in 1974 dollars.

The Carnegie plans would increase FTE enrollments by about 13

percent, the Equal Opportunity Plan II by about 6 percent, the Equal Opportunity Plan I by about 4 percent, and the CED plan by less than 4 percent.

The plans would not be equally easy to administer and implement. With the most difficult task of obtaining funds completed, the Carnegie plans could be administered through existing college and university aid offices. The CED plan, on the other hand, would require national direction or legislation to require the state-controlled colleges and universities to raise their tuitions at the proper time and by the required amounts. The Equal Opportunity plans would require regulation of some type to prevent unwarranted tuition increases, which otherwise would be partially covered by grants.

Let us remind ourselves all the plans induce increase in the demand for higher education. We have implicitly assumed that there would be a sufficiently large supply of places so that this increased demand could be satisfied without either increasing instructional cost per student or rationing places. In cases where the implementation of a plan would induce greater demand than could be accommodated, actual attendance would be less than we have predicted. This means that our cost estimates should be considered an upper limit; we do not expect the plans to cost more than our estimates, and in some cases they may cost less.

It is hardly conceivable that in the near future the people of the United States will turn away from college and university education

as a means of enriching their lives and enhancing their economic opportunities. As long as higher education is perceived as necessary to attain prestige and pecuniary reward, public policies will be needed to achieve social and exonomic justice. Our work is a step toward understanding and evaluating the efficacy of policy proposals. It is not a perfect tool.

Our model would be strengthened if we had sufficient data to statistically estimate the transition probabilities and, better still, to estimate them as functions of interesting variables such as academic ability and costs of attendance. The effects of tuition changes on current full- and part-time students are far less well investigated than their effects on high school seniors. In fact, there is almost no hard information about any type of student other than prospective freshmen. Further research may show that disaggregation by race or sex, or by both race and sex, gives qualitatively different results than does our aggregated model. How the policy plans we have analyzed would affect the adult population is open to speculation and should be investigated.

VIII. Appendices

Appendix I

The Radner-Miller Model, Its Adaptation to our Model and that of the NCFPE

Radner and Miller define

$$(1) \qquad f_{ij} = \gamma \frac{C_j}{Y_i} + \alpha A_i S_j \qquad i=1,I \qquad j=i,J$$

where

C_j = The cost of the j^{th} IHE alternative

Y_i = Family income of the i^{th} student

A_i = Academic Ability of the i^{th} student (SAT scores)

S_j = Selectivity of the j^{th} IHE alternative (Representative SAT score)

γ and α are parameters to be estimated.

Then define

$$F_{ij} = e^{f_{ij}}$$

then

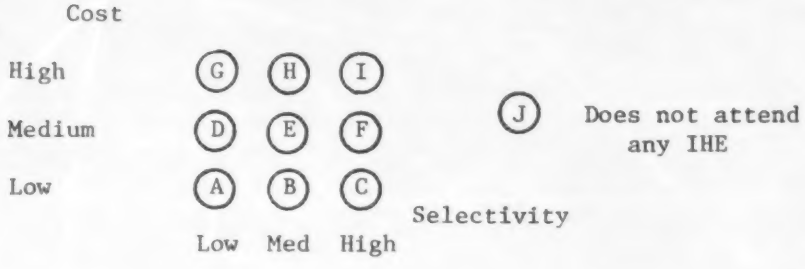
$$(2) \qquad P_{ik} = \frac{F_{ik}}{\sum_{j \epsilon J} F_{ij}}$$

and P_{ik} is the probability that student type i chooses IHE alternative j from the set of alternatives J. The set J includes the alternative of not going at all and is generally restricted to IHEs with selectivity commensurate with student i's ability.

The γ and α were estimated for several levels of ability. With these estimates, (2) was evaluated as a function of C_j to obtain predicted probabilities for the i^{th} type student.

Radner-Miller's freshman demand model specifies a choice set of nine types of institutions of higher education and a tenth option of not attending any IHE. The types of IHEs are categorized by the cost of attending the IHE and the selectivity of the IHE with respect to student academic ability.

TABLE I



As seen in Table I, IHE type E represents a medium-cost, medium-selectivity IHE.

Variations in cost and selectivity are one way of describing IHEs. The Radner-Miller cost-selectivity categories are roughly equivalent to the descriptions in Table II.

TABLE II

A. Public community colleges
B. Public state universities
C. Public universities
D. Trade schools and private Junior colleges
E. Public state colleges
F. Public universities
G. Private four-year colleges and universities
H. Private four-year colleges and universities
I. Private four-year colleges and universities

For comparison the NCFPE IHE categories are presented in Table III. Table IV shows the manner in which Radner-Miller categories have been transferred to NCFPE categories.

TABLE III

NCFPE IHE types

1. Public two-year
2. Public four-year; lower division
3. Public four-year; upper division
4. Public four-year; graduate
5. Private two-year
6. Private four-year; lower division
7. Private four-year; upper division
8. Private four-year; graduate
9. Non-collegiate

TABLE IV

NCFPE Type and Radner-Miller Equivalent

1.	A
2., 3., 4.	each equals the average of B, C, E, and F
5.	D
6., 7., 8.	each equals the average of G, H, and I
9.	D

The NCFPE make two important assumptions in transferring the Radner-Miller institution types (and the price responses associated with them) to their own. First, they assume that all students react equally to cost changes regardless of academic standing. Second, they assume that all students and potential freshmen react the same way to price changes. The data to support or refute these assumptions are not available.

Appendix II

NCFPE Price-Response Coefficients

From the Radner-Miller model we have;

$$\frac{\delta \phi_{ij}}{\delta C_{ij}} / \phi_{ij} = (\beta_i / Y_i)(1 - \phi_{ij}) \qquad (1)$$

$$\frac{\delta \phi_{ik}}{\delta C_{ij}} / \phi_{ik} = (\beta_i / Y_i)\, \phi_{ij} \qquad (2)$$

(1) and (2) are the direct and indirect price-response coefficients respectively. (1) is a measure of the change in the proportion of students of type i going to IHE type j given a change in the cost of j. The ϕ_{ij} are the predicted relative frequencies of students of type i attending IHE j. A student of type i has family income Y_i, and β_i, an estimate of the coefficient of the burden of the cost of attending from the Radner-Miller model and a measure of academic ability, A_i.

The NCFPE cannot use the ϕ_{ij} and β_i directly, as these variables differ by student ability. From the Radner-Miller raw data set (SCOPE) we have:

W_{ik} = proportion of students with income level i having academic ability in the k^{th} quartile.

The NCFPE use three family income classifications in calculating the price-response coefficient.

Low income = Income less than $7,500

Middle income = Income less than $15,000 but greater than or equal to $7,500

High income = Income greater than or equal to $15,000

Using the weights W_{ik} the NCFPE calculates

$$\bar{\beta}_i = \sum_k W_{ik} \times \beta_k$$

and

$$\bar{\phi}_{ij} = \sum W_{ik} \phi_{ijk}$$

$i = 1,3$ income types,

$j = 1,9$ IHE types

β_1, for example, represents the coefficient of the burden of cost for a student of low family income and average, among other low-income students, academic ability. $\bar{\phi}_{23}$ represents the predicted frequency of a student in the middle income range, of average academic ability for the middle income range, going to IHE type 3.

The values of Y_i, income evaluation points, that the NCFPE use are $6,000 (low income), $12,000 (middle income), and $18,000 (high income). These values are not SCOPE sample means. The values of Y_i were apparently chosen for computation ease and a lack of any better estimate.

The NCFPE price-response coefficients were actually calculated as

$$PRC_{Direct}^{i,j} = \frac{\bar{\beta}_i}{Y_i}(1 - \bar{\phi}_{ij}) \qquad (3)$$

$$PRC_{Indirect}^{i,k} = \frac{\bar{\beta}_i}{Y_i}\bar{\phi}_{ij} \qquad (4)$$

$i = 1,3$ income groups

$j = 1,9$ IHE types

Finally, (3) and (4) were multiplied by 100 to reflect a cost change of $100 and again by 100 to represent percentages. The NCFPE made an error in calculating the indirect price response of low income level students to IHE category H(i=8) . Instead of the $.10 \times 10^{-4}$ value they report, the correct value is $.21 \times 10^{-4}$.

The NCFPE price response-coefficients represent the price responses of average (over academic ability) students. We have calculated a simple alternative which represents the average price response of four academic-type students within an income group:

$$PRC^*(Direct)_{i,j} = \sum_{k=1}^{4} W_{ik} \frac{\beta_{ik}}{Y_i} (1 - \phi_{ik}) \qquad (5)$$

$$PRC^*(Indirect)_{i,k} = \sum_{k=1}^{4} W_{ik} \frac{\beta_{ik}}{Y_i} \phi_{ijk} \qquad (6)$$

where the W_{ik} are, as noted above, the income-academic ability frequencies from Radner-Miller's data set. The NCFPE price-response coefficients are shown in Table V. Our first alternative price-response coefficients, (5) and (6), are in Table VI.

Tables VII and VIII show two more sets of price-response coefficients derived from different data. Using the same procedure as in Radner-Miller chapter 4, we have calculated the expected demand distribution with the NCFPE income ranges. Rather than producing the expected demand distribution from the national population, we calculate demand distributions for three income groups, low (income less than $7,500),

middle (income between $15,000 and $7,499), and high (income above $14,999). Table VII shows the price response coefficients calculated from these expected demand distributions. The β_i are the same as those used by the NCFPE:

$$PRC^{**}(\text{Direct})_{i,j} = \frac{\bar{\beta}_i}{Y_i}(1 - \phi^*_{ij})$$

$$PRC^{**}(\text{Indirect})_{i,k} = \frac{\bar{\beta}_i}{Y_i}\phi^*_{ij}$$

Table VIII differs from Table VII in that the β_i used are weighted by the same bivariate normal density function as has been used to calculate the $\phi^*_{i,j}$.

$$PRC^{***}(\text{Direct})_{i,j} = \frac{\beta_i}{Y_i}(1 - \phi^*_{ij})$$

$$PRC^{***}(\text{Indirect})_{i,k} = \frac{\beta_i}{Y_i}\phi^*_{ij}$$

TABLE V

NCFPE price response coefficients*

Low Income (Family income < $7,500)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-2.95	0.32	0.00	0.00	0.22	0.20	0.00	0.00	0.22
2	0.51	3.14	0.00	0.00	0.22	0.20	0.00	0.00	0.22
3	0.00	0.00	-3.14	0.00	0.00	0.00	0.20	0.00	0.00
4	0.00	0.00	0.00	-3.14	0.00	0.00	0.00	0.20	0.00
5	0.51	0.32	0.00	0.00	-3.24	0.20	0.00	0.00	0.22
6	0.51	0.32	0.00	0.00	0.22	-3.26	0.00	0.00	0.22
7	0.00	0.00	0.32	0.00	0.00	0.00	-3.26	0.00	0.00
8	0.00	0.00	0.00	0.32	0.00	0.00	0.00	-3.26	0.00
9	0.51	0.32	0.00	0.00	0.22	0.20	0.00	0.00	-3.24

Middle Income
(Family income between $7,500 and $14,999)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-1.23	0.15	0.00	0.00	0.09	0.13	0.00	0.00	0.09
2	0.13	-1.21	0.00	0.00	0.09	0.13	0.00	0.00	0.09
3	0.00	0.00	-1.21	0.00	0.00	0.00	0.13	0.00	0.00
4	0.00	0.00	0.00	-1.21	0.00	0.00	0.00	0.13	0.00
5	0.13	0.15	0.00	0.00	-1.27	0.13	0.00	0.00	0.09
6	0.13	0.15	0.00	0.00	0.09	-1.24	0.00	0.00	0.09
7	0.00	0.00	0.15	0.00	0.00	0.00	-1.24	0.00	0.00
8	0.00	0.00	0.00	0.15	0.00	0.00	0.00	-1.24	0.00
9	0.13	0.15	0.00	0.00	0.09	0.13	0.00	0.00	-1.27

High Income (Family income ≥ $15,000)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-0.75	0.09	0.00	0.00	0.05	0.09	0.00	0.00	0.05
2	0.06	-0.71	0.00	0.00	0.05	0.09	0.00	0.00	0.05
3	0.00	0.00	-0.71	0.00	0.00	0.00	0.09	0.00	0.00
4	0.00	0.00	0.00	-0.71	0.00	0.00	0.00	0.09	0.00
5	0.06	0.09	0.00	0.00	-0.76	0.09	0.00	0.00	0.05
6	0.06	0.09	0.00	0.00	0.05	-0.72	0.00	0.00	0.05
7	0.00	0.00	0.09	0.00	0.00	0.00	-0.72	0.00	0.00
8	0.00	0.00	0.00	0.09	0.00	0.00	0.00	-0.72	0.00
9	0.06	0.09	0.00	0.00	0.05	0.09	0.00	0.00	-0.76

*Represent the percentage change in enrollment in the IHE in the left column given a $100 price increase in the IHEs as listed in the top row.

TABLE VI

Price Response Coefficients Representing the Average Price Response of Four Ability Quartiles within each Income Group*

Low Income (Family income < $7,500)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-2.77	0.21	0.00	0.00	0.27	0.12	0.00	0.00	0.27
2	0.69	-1.13	0.00	0.00	0.27	0.12	0.00	0.00	0.27
3	0.00	0.00	-1.13	0.00	0.00	0.00	0.12	0.00	0.00
4	0.00	0.00	0.00	-1.13	0.00	0.00	0.00	0.12	0.00
5	0.69	0.21	0.00	0.00	-3.19	0.12	0.00	0.00	0.27
6	0.69	0.21	0.00	0.00	0.27	-1.26	0.00	0.00	0.27
7	0.00	0.00	0.21	0.00	0.00	0.00	-1.26	0.00	0.00
8	0.00	0.00	0.00	0.21	0.00	0.00	0.00	-1.26	0.00
9	0.69	0.21	0.00	0.00	0.27	0.12	0.00	0.00	-3.19

Middle Income
(Family income between $7,500 and $14,999)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-1.17	0.11	0.00	0.00	0.13	0.09	0.00	0.00	0.13
2	0.20	-0.68	0.00	0.00	0.13	0.09	0.00	0.00	0.13
3	0.00	0.00	-0.68	0.00	0.00	0.00	0.09	0.00	0.00
4	0.00	0.00	0.00	-0.68	0.00	0.00	0.00	0.09	0.00
5	0.20	0.11	0.00	0.00	-1.24	0.09	0.00	0.00	0.13
6	0.20	0.11	0.00	0.00	0.13	-0.70	0.00	0.00	0.13
7	0.00	0.00	0.11	0.00	0.00	0.00	-0.70	0.00	0.00
8	0.00	0.00	0.00	0.11	0.00	0.00	0.00	-0.70	0.00
9	0.20	0.11	0.00	0.00	0.13	0.09	0.00	0.00	-1.24

High Income (Family income $\geq$ $15,000)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-0.71	0.07	0.00	0.00	0.07	0.07	0.00	0.00	0.07
2	0.09	-0.47	0.00	0.00	0.07	0.07	0.00	0.00	0.07
3	0.00	0.00	-0.47	0.00	0.00	0.00	0.07	0.00	0.00
4	0.00	0.00	0.00	-0.47	0.00	0.00	0.00	0.07	0.00
5	0.09	0.07	0.00	0.00	-0.74	0.07	0.00	0.00	0.07
6	0.09	0.07	0.00	0.00	0.07	-0.46	0.00	0.00	0.07
7	0.00	0.00	0.07	0.00	0.00	0.00	-0.46	0.00	0.00
8	0.00	0.00	0.00	0.07	0.00	0.00	0.00	-0.46	0.00
9	0.09	0.07	0.00	0.00	0.07	0.07	0.00	0.00	-0.74

*Represent the percentage change in enrollment in the IHE in the left column given a $100 price increase in the IHEs as listed in the top row.

TABLE VII

Price Response Coefficient Derived from Expected Demand Distribution using NCFPE β_1*

Low Income (Family income < $7,500)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-2.66	0.12	0.00	0.00	0.12	0.06	0.00	0.00	0.12
2	0.80	-3.34	0.00	0.00	0.12	0.06	0.00	0.00	0.12
3	0.00	0.00	-3.34	0.00	0.00	0.00	0.06	0.00	0.00
4	0.00	0.00	0.00	-3.34	0.00	0.00	0.00	0.06	0.00
5	0.80	0.12	0.00	0.00	-3.34	0.06	0.00	0.00	0.12
6	0.80	0.12	0.00	0.00	0.12	-3.40	0.00	0.00	0.12
7	0.00	0.00	0.12	0.00	0.00	0.00	-3.40	0.00	0.00
8	0.00	0.00	0.00	0.12	0.00	0.00	0.00	-3.40	0.00
9	0.80	0.12	0.00	0.00	0.12	0.06	0.00	0.00	-3.34

Middle Income
(Family income between $7,500 and $14,999)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-1.08	0.05	0.00	0.00	0.18	0.06	0.00	0.00	0.18
2	0.28	-1.31	0.00	0.00	0.18	0.06	0.00	0.00	0.18
3	0.00	0.00	-1.31	0.00	0.00	0.00	0.06	0.00	0.00
4	0.00	0.00	0.00	-1.31	0.00	0.00	0.00	0.06	0.00
5	0.28	0.05	0.00	0.00	-1.19	0.06	0.00	0.00	0.18
6	0.28	0.05	0.00	0.00	0.18	-1.30	0.00	0.00	0.18
7	0.00	0.00	0.05	0.00	0.00	0.00	-1.30	0.00	0.00
8	0.00	0.00	0.00	0.05	0.00	0.00	0.00	-1.30	0.00
9	0.28	0.05	0.00	0.00	0.18	0.06	0.00	0.00	-1.19

High Income (Family income $\geq$ $15,000)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-0.65	0.15	0.00	0.00	0.15	0.15	0.00	0.00	0.15
2	0.03	-0.77	0.00	0.00	0.03	0.03	0.00	0.00	0.03
3	0.00	0.00	-0.77	0.00	0.00	0.00	0.03	0.00	0.00
4	0.00	0.00	0.00	-0.77	0.00	0.00	0.00	0.03	0.00
5	0.13	0.13	0.00	0.00	-0.68	0.13	0.00	0.00	0.13
6	0.05	0.05	0.00	0.00	0.05	-0.76	0.00	0.00	0.05
7	0.00	0.00	0.05	0.00	0.00	0.00	-0.76	0.00	0.00
8	0.00	0.00	0.00	0.05	0.00	0.00	0.00	-0.76	0.00
9	0.13	0.13	0.00	0.00	0.13	0.13	0.00	0.00	-0.68

*Represent the percentage change in enrollment in the IHE in the left column given a $100 price increase in the IHEs as listed in the top row.

TABLE VIII

Price Response Coefficient Derived from Expected Demand Distribution Using Bivariate Normal Weighted β_i*

Low Income (Family income < $7,500)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-3.71	0.17	0.00	0.00	0.17	0.08	0.00	0.00	0.17
2	1.12	-4.66	0.00	0.00	0.17	0.08	0.00	0.00	0.17
3	0.00	0.00	-4.66	0.00	0.00	0.00	0.08	0.00	0.00
4	0.00	0.00	0.00	-4.66	0.00	0.00	0.00	0.08	0.00
5	1.12	0.17	0.00	0.00	-4.66	0.08	0.00	0.00	0.17
6	1.12	0.17	0.00	0.00	0.17	-4.75	0.00	0.00	0.17
7	0.00	0.00	0.17	0.00	0.00	0.00	-4.75	0.00	0.00
8	0.00	0.00	0.00	0.17	0.00	0.00	0.00	-4.75	0.00
9	1.12	0.17	0.00	0.00	0.17	0.08	0.00	0.00	-4.66

Middle Income
(Family income between $7,500 and $14,999)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-1.87	0.09	0.00	0.00	0.31	0.11	0.00	0.00	0.31
2	0.49	-2.27	0.00	0.00	0.31	0.11	0.00	0.00	0.31
3	0.00	0.00	-2.27	0.00	0.00	0.00	0.11	0.00	0.00
4	0.00	0.00	0.00	-2.27	0.00	0.00	0.00	0.11	0.00
5	0.49	0.09	0.00	0.00	-2.06	0.11	0.00	0.00	0.31
6	0.49	0.09	0.00	0.00	0.31	-2.26	0.00	0.00	0.31
7	0.00	0.00	0.09	0.00	0.00	0.00	-2.26	0.00	0.00
8	0.00	0.00	0.00	0.09	0.00	0.00	0.00	-2.26	0.00
9	0.49	0.09	0.00	0.00	0.31	0.11	0.00	0.00	-2.06

High Income (Family income $\geq$ $15,000)

Institutional Category	1	2	3	4	5	6	7	8	9
1	-1.27	0.06	0.00	0.00	0.25	0.09	0.00	0.00	0.25
2	0.30	-1.51	0.00	0.00	0.25	0.09	0.00	0.00	0.25
3	0.00	0.00	-1.51	0.00	0.00	0.00	0.09	0.00	0.00
4	0.00	0.00	0.00	-1.51	0.00	0.00	0.00	0.09	0.00
5	0.30	0.06	0.00	0.00	-1.32	0.09	0.00	0.00	0.25
6	0.30	0.06	0.00	0.00	0.25	-1.48	0.00	0.00	0.25
7	0.00	0.00	0.06	0.00	0.00	0.00	-1.48	0.00	0.00
8	0.00	0.00	0.00	0.06	0.00	0.00	0.00	-1.48	0.00
9	0.30	0.06	0.00	0.00	0.25	0.09	0.00	0.00	-1.32

*Represent the percentage change in enrollment in the IHE in the left column given a $100 price increase in the IHEs as listed in the top row.